**Paper Reference 9MA0/01**
**Pearson Edexcel**
**Level 3 GCE**

# Mathematics

## Advanced

## PAPER 1: Pure Mathematics 1

**Tuesday 6 June 2023 – Afternoon**

**Time: 2 hours**

**YOU MUST HAVE**
**Mathematical Formulae and Statistical Tables (Green), calculator**

**YOU WILL BE GIVEN**
**Diagram Booklet**
**Answer Booklet**

**Y72804A**

Pearson

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

# INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or on the separate diagrams – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

# INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 15 questions in this Question Paper.
The total mark for this paper is 100.

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There may be spare copies of some diagrams in case you need them.

## ADVICE

**Read each question carefully before you start to answer it.**

**Try to answer every question.**

**Check your answers if you have time at the end.**

1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,dx$$

writing each term in simplest form.

**(Total for Question 1 is 4 marks)**

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Turn over

2. **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \qquad x \in \mathbb{R}$$

where $a$ is a positive constant.

(continued on the next page)

2. continued.

Given $(x - a)$ is a factor of $f(x)$,

(a) show that

$$a(4a^2 + 5a - 6) = 0$$

(2 marks)

(continued on the next page)

2. continued.

(b) Hence

(i) find the value of $a$

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3$$

(4 marks)

(Total for Question 2 is 6 marks)

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Turn over

3. Relative to a fixed origin $O$

- the point $A$ has position vector $5\underline{i} + 3\underline{j} + 2\underline{k}$
- the point $B$ has position vector $2\underline{i} + 4\underline{j} + a\underline{k}$

where $a$ is a positive integer.

(a) Show that

$$|\overrightarrow{OA}| = \sqrt{38}$$

(1 mark)

(continued on the next page)

3. continued.

(b) Find the smallest value of $a$ for which

$$|\overrightarrow{OB}| > |\overrightarrow{OA}|$$

(2 marks)

**(Total for Question 3 is 3 marks)**

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Turn over

**4.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

**(continued on the next page)**

4. continued.

The curve $C$ has equation

$y = f(x)$ where $x \in \mathbb{R}$

Given that

- $f'(x) = 2x + \dfrac{1}{2}\cos x$
- the curve has a stationary point with $x$ coordinate $\alpha$
- $\alpha$ is small

(continued on the next page)

4. continued.

(a) use the small angle approximation for $\cos x$ to estimate the value of $\alpha$ to 3 decimal places.

(3 marks)

(continued on the next page)

4. continued.

The point

$P(0, 3)$ lies on $C$

(b) Find the equation of the tangent to the curve at $P$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be found.
(2 marks)

(Total for Question 4 is 5 marks)

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5. Refer to the table for Question 5 in the Diagram Booklet.

A continuous curve has equation

$y = f(x)$

The table in the Diagram Booklet shows corresponding values of $x$ and $y$ for this curve, where $a$ and $b$ are constants.

(continued on the next page)

**5. continued.**

**The trapezium rule is used, with all the $y$ values in the table, to find an approximate area under the curve between**

**$x = 3$ and**

**$x = 4$**

**Given that this area is $17 \cdot 59$**

**(a) show that**

$$a + 2b = 51$$

**(3 marks)**

**(continued on the next page)**

**5. continued.**

**Given also that the sum of all the $y$ values in the table is $97 \cdot 2$**

**(b) find the value of $a$ and the value of $b$**

**(3 marks)**

**(Total for Question 5 is 6 marks)**

6. $a = \log_2 x$

$b = \log_2(x + 8)$

Express in terms of $a$ and/or $b$

(a) $\log_2 \sqrt{x}$

(1 mark)

(b) $\log_2(x^2 + 8x)$

(2 marks)

(continued on the next page)

**6. continued.**

**(c)** $\log_2\left(8 + \frac{64}{x}\right)$

**Give your answer in simplest form.**

**(3 marks)**

**(Total for Question 6 is 6 marks)**

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7. The function $f$ is defined by

$$f(x) = 3 + \sqrt{x - 2} \qquad x \in \mathbb{R} \qquad x > 2$$

(a) State the range of $f$

(1 mark)

(continued on the next page)

7. continued.

(b) Find $f^{-1}$

(3 marks)

The function $g$ is defined by

$$g(x) = \frac{15}{x-3} \qquad x \in \mathbb{R} \qquad x \neq 3$$

(c) Find

$gf(6)$

(2 marks)

(continued on the next page)

**7. continued.**

**(d)** **Find the exact value of the constant $a$ for which**

$$f(a^2 + 2) = g(a)$$

**(2 marks)**

**(Total for Question 7 is 8 marks)**

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8. Refer to the diagram for Question 8 in the Diagram Booklet.
It is NOT accurately drawn.
It shows the plan view of a stage.

The plan view shows two congruent triangles ABO and GFO joined to a sector OCDEO of a circle, centre O, where

- angle COE = $2 \cdot 3$ radians
- arc length CDE = $27 \cdot 6$ metres
- AOG is a straight line of length 15 metres

(continued on the next page)

8. continued.

(a) Show that

$OC = 12$ metres

(2 marks)

(b) Show that the size of angle AOB is $0 \cdot 421$ radians correct to 3 decimal places.

(2 marks)

(continued on the next page)

**8. continued.**

**Given that the total length of the front of the stage, BCDEF, is 35 metres,**

**(c) find the total area of the stage, giving your answer to the nearest square metre.**

**(6 marks)**

**(Total for Question 8 is 10 marks)**

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9. The first three terms of a geometric sequence are

$3k + 4 \qquad 12 - 3k \qquad k + 16$

where $k$ is a constant.

(a) Show that $k$ satisfies the equation

$$3k^2 - 62k + 40 = 0$$

(2 marks)

(continued on the next page)

Turn over

9. continued.

Given that the sequence converges,

(b) (i) find the value of $k$, giving a reason for your answer,

(ii) find the value of $S_{\infty}$

(5 marks)

(Total for Question 9 is 7 marks)

10. A circle C has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where $k$ is a constant.

(a) Find in terms of $k$,

(i) the coordinates of the centre of C

(ii) the radius of C

(3 marks)

(continued on the next page)

Turn over

**10. continued.**

**The line with equation**

**$y = 2x - 1$ intersects $C$ at**

**$2$ distinct points.**

**(b) Find the range of possible values of $k$**

**(6 marks)**

**(Total for Question 10 is 9 marks)**

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**11. Refer to the diagram for Question 11 in the Diagram Booklet.**

**The value, $V$ pounds, of a mobile phone, $t$ months after it was bought, is modelled by**

$$V = ab^t$$

**where $a$ and $b$ are constants.**

**The diagram shows the linear relationship between $\log_{10}V$ and $t$**

**The line passes through the points $(0, 3)$ and $(10, 2\cdot79)$ as shown.**

**(continued on the next page)**

**11. continued.**

**Using these points,**

**(a) find the initial value of the phone,**

**(2 marks)**

**(b) find a complete equation for $V$ in terms of $t$, giving the exact value of $a$ and giving the value of $b$ to 3 significant figures.**

**(3 marks)**

**(continued on the next page)**

**11. continued.**

**Exactly 2 years after it was bought, the value of the phone was £320**

**(c) Use this information to evaluate the reliability of the model.**

**(2 marks)**

**(Total for Question 11 is 7 marks)**

12. $y = \sin x$

where $x$ is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

(continued on the next page)

**12. continued.**

**You may**

- **use without proof the formula for $\sin(A \pm B)$**
- **assume that as**

  **$h \longrightarrow 0$,**

  $$\frac{\sin h}{h} \longrightarrow 1 \text{ and}$$

  $$\frac{\cos h - 1}{h} \longrightarrow 0$$

**(Total for Question 12 is 5 marks)**

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**13. On a roller coaster ride, passengers travel in carriages around a track.**

**On the ride, carriages complete multiple circuits of the track such that**

- **the maximum vertical height of a carriage above the ground is 60 metres**
- **a carriage starts a circuit at a vertical height of 2 metres above the ground**
- **the ground is horizontal**

**(continued on the next page)**

13. continued.

The vertical height, $H$ metres, of a carriage above the ground, $t$ seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where $a$ and $b$ are positive constants.

(continued on the next page)

13. continued.

(a) Find a complete equation for the model.

(3 marks)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$

(1 mark)

(continued on the next page)

**13. continued.**

**In an alternative model, the vertical height, $H$ metres, of a carriage above the ground, $t$ seconds after the carriage starts the first circuit, is given by**

$$H = 29\cos(9t + \alpha)^\circ + \beta$$

$$0 \leq \alpha < 360^\circ$$

**where $\alpha$ and $\beta$ are constants.**

**(continued on the next page)**

**13. continued.**

**(c) Find a complete equation for the alternative model.**

**(2 marks)**

**Given that the carriage moves continuously for 2 minutes,**

**(d) give a reason why the alternative model would be more appropriate.**

**(1 mark)**

**(Total for Question 13 is 7 marks)**

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14. Prove, using algebra, that

$$(n + 1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

**(Total for Question 14 is 4 marks)**

**15. A curve has equation**

**$y = f(x)$, where**

$$f(x) = \frac{7xe^{x}}{\sqrt{e^{3x} - 2}} \qquad x > \ln\sqrt[3]{2}$$

**(a) Show that**

$$f'(x) = \frac{7e^{x}(e^{3x}(2 - x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

**where A and B are constants to be found.**

**(5 marks)**

(continued on the next page)

15. continued.

(b) Hence show that the $x$ coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2 marks)

(continued on the next page)

15. continued.

The equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$ has two positive roots

$\alpha$ and $\beta$

where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for $\alpha$ and $\beta$

(continued on the next page)

Turn over

**15. continued.**

**Refer to the diagram for Question 15(c) in the Diagram Booklet.**

**It shows a plot of part of the curve with equation**

$$y = \frac{2e^{3x} - 4}{e^{3x} + 4}$$ **and part of the line**

**with equation** $y = x$

**(continued on the next page)**

15. continued.

Using the diagram in the Diagram Booklet,

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for $\beta$

(1 mark)

(continued on the next page)

15. continued.

Use the iteration formula with $x_1 = 1$,

to find, to 3 decimal places,

(d) (i) the value of $x_2$

(ii) the value of $\beta$

(3 marks)

(continued on the next page)

**15. continued.**

**Using a suitable interval and a suitable function that should be stated**

**(e) show that**

**$\alpha = 0 \cdot 432$ to 3 decimal places.**

**(2 marks)**

**(Total for Question 15 is 13 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**

**END OF PAPER**

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